

Quantum Information Processing Without Joint Measurement

Ming Yang^{1,*} and Zhuo-Liang Cao^{1,†}

¹*School of Physics & Material Science, Anhui University, Hefei, 230039, People's Republic of China*

We propose a linear optical scheme for the teleportation of unknown ionic states, the entanglement concentration for nonmaximally entangled states for ions via entanglement swapping and the remote preparation for ionic entangled states. The unique advantage of the scheme is that the joint Bell-state measurement needed in the previous schemes is not needed in the current scheme, i.e. the joint Bell-state measurement has been converted into the product of separate measurements on single ions and photons. In addition, the current scheme can realize the quantum information processes for ions by using linear optical elements, which simplify the implementation of quantum information processing for ions.

PACS numbers: 03.67.Hk, 03.67.Mn, 03.67.Pp

I. INTRODUCTION

Joint measurements play an important role in quantum information processing (QIP). In quantum teleportation, an unknown quantum state will be sent from sender to receiver via a quantum channel with the help of classical communication. During this process, the sender will operate a joint Bell-state measurement on the unknown-state particle and one of the entangled particles she possesses [1]. In entanglement swapping, there are usually three spatially separate users, and two of them have shared one pair of entangled particles with the third user. Then the third user will operate a joint Bell-state measurement on the two particles he possesses. Corresponding to the measurement result, the two particles possessed by the two spatially separate users will collapse into an entangled state without any entanglement before the joint measurement [2]. In remote state preparation [3], if we want to prepare some entangled state remotely via two pairs of entangled particles as the quantum channel, the joint measurement is also a necessity [4]. Like the above-mentioned entanglement swapping, there are usually three spatially separate users, and two of them have shared one pair of entangled particles with the third user. Then the third user will operate a special type of joint measurement, which corresponds to the entangled state the third user want to prepare remotely, on the two particles he possesses. After the third user informs the first or the second user the measurement results, the particles of the first and the second user are prepared in the entangled state that the third user want to prepare remotely. In addition, joint measurements are also needed in the quantum dense coding [5]. The sender and the receiver share an entangled pairs, and the sender will encode the 2-bit classical information on the entangled particle he possesses by operating four possible single-bit operations on the particle. Then this

particle(one qubit) will be sent to the receiver. The receiver will operate a joint Bell-state measurement on the two particles to decode the 2-bit classical information the sender encoded.

Form the above analysis, joint measurements are needed in quantum teleportation, entanglement swapping, remote preparation of entangled state via two pairs of entangled particles as the quantum channel and quantum dense coding, etc. But it is very difficult to realize joint measurements in experiment. Usually, joint measurements will be converted into the product of separate measurements on single particle [6, 7, 8, 9]. For the photon case, teleportation of unknown polarization state of photons has been realized in experiment [10], where the joint Bell-state measurement has been converted into the product of separate measurements on single photon by using the linear optical elements, such as polarization beam splitters and photon detectors. But in this experimental scheme, the four Bell states can not be discriminated conclusively and completely. So Zhao *et al* proposed an alternative scheme for the conclusive discrimination of the four Bell states of photons with the help of the ancillary entangled pairs of photons [8]. In cavity QED domain, the Bell-state measurement on atoms has been converted into the separate measurements on single atom by using the controlled-NOT (C-NOT) gate operations [6], where the dispersive interaction between two atoms and a cavity mode plays an important role. To avoid the difficulty of the C-NOT gate operations, Zheng and Ye all proposed the schemes to teleport an unknown atomic state without Bell-state measurement, and the interaction between atoms and cavity modes decomposes the Bell states into product states.

From the experimental point of view, Bell-state measurements have been realized for the photon case. However, because it is difficult to realize Bell-state measurements for atomic (ionic) states in experiment [11, 12], the implementation of quantum teleportation [11, 12], entanglement swapping, remote preparation of entangled state and quantum dense coding for atomic (ionic) states are all not easy. In our previous contribution, we have proposed the entanglement swapping scheme for atomic system without joint measurement [13, 14], where the in-

*Electronic address: mingyang@ahu.edu.cn

†Electronic address: zlcao@ahu.edu.cn

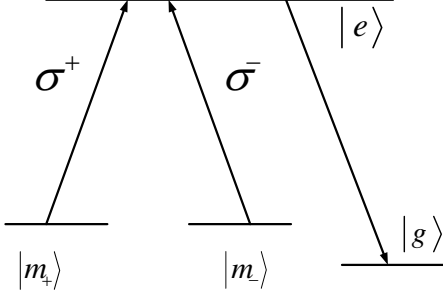


FIG. 1: Level configuration of the ions used in the scheme. The ions, which are in the degenerate states $|m_+\rangle$ and $|m_-\rangle$, can be excited into the unstable excited state $|e\rangle$ by absorbing one σ^+ or σ^- polarized photon, then it can decay to the stable ground state $|g\rangle$ with a scattered photon rapidly.

teraction (resonant and nonresonant cases) between atoms and cavity modes replaces the Bell-state analyzer. But, due to the complexity of the cavity QED techniques, it is difficult to realize these schemes in experiment. So, in this paper, we will propose an alternative scheme for some quantum information processes for ions, such as, quantum teleportation, entanglement concentration via entanglement swapping and remote preparation of entangled states, etc. We will use the linear optical elements. The main setup of the scheme is a Mach-Zehnder interferometers (MZI) with two ions placed on the two arms of the MZI. We can decide whether the scheme succeed or not by operating single photon measurements on the two output ports of the MZI. The unique advantage of the current scheme is that the quantum information processing for ions can be realized by using linear optical elements, which decreases the difficulty of the experimental implementation. In addition, by using the MZI, the joint measurements are all decomposed into single photon measurements and single ion measurements, so the current scheme avoids the difficulty of realizing joint measurements.

II. QUANTUM TELEPORTATION OF UNKNOWN IONIC STATES VIA LINEAR OPTICS

Consider the three-level ionic system, where $|m_+\rangle$ and $|m_-\rangle$ are two degenerate metastable states of ions, and $|e\rangle$ is the excited state. The level configuration of the ions is depicted in Fig.1. The ions can be excited from $|m_+\rangle$ or $|m_-\rangle$ to the excited states $|e\rangle$ by absorbing one σ^+ or σ^- circular polarization photon with unit efficiency. Because the excited state $|e\rangle$ is not a stable one, the ions in that state will decay rapidly to the stable ground state $|g\rangle$ with a scattered photon $|S\rangle$.

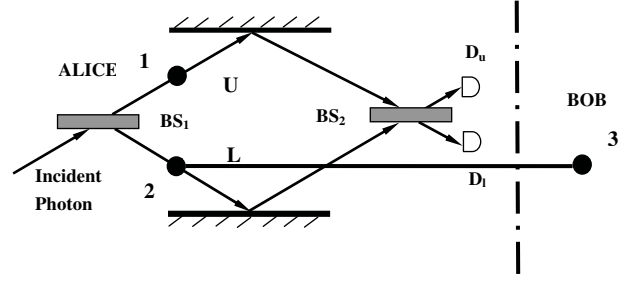


FIG. 2: The setup for teleportation scheme. Alice places ions 1, 2 on the upper arm and lower arm of the MZI, respectively. One σ^+ polarized photon will be superimposed on the first beam splitter (BS₁). After the BS₁ the photon will take two possible paths (u denotes the upper path and l denotes the lower one). Reflected by two mirrors, the two possible paths will be recombined at the second beam splitter (BS₂). Single photon detection will be operated on the two output ports of the MZI after the BS₂. If the right lower output port detector (D_l) fires, the teleportation can succeed.

This process can be expressed as [15, 16]:

$$a_{\pm}^+|0\rangle|m_{\pm}\rangle \longrightarrow |S\rangle|g\rangle. \quad (1)$$

Suppose the unknown state of ion 1 to be teleported is:

$$|\Psi\rangle_1 = \alpha|m_+\rangle_1 + \beta|m_-\rangle_1, \quad (2)$$

where α, β are normalization coefficients, and $|\alpha|^2 + |\beta|^2 = 1$. Without loss of generality, we can assume that α, β are all real numbers.

Before teleportation, the sender (Alice) and the receiver (Bob) share a maximally entangled pair of ions 2, 3:

$$|\Psi\rangle_{23} = \frac{1}{\sqrt{2}}(|m_+\rangle_2|m_+\rangle_3 + |m_-\rangle_2|m_-\rangle_3). \quad (3)$$

Alice possesses the ions 1, 2, and Bob has access to ion 3. To complete the teleportation of unknown state of ion 1, a MZI will be introduced at Alice's location. The ions 1, 2 will be placed on upper arm and lower arm of the MZI, respectively, by using trapping technology [17]. One σ^+ polarized photon will be superimposed on the MZI from the left lower input port. The main setup is depicted in Fig.2. The effect of the BS on the input photon can be expressed as:

$$a_{l,\pm}^+|0\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(a_{u,\pm}^+ + ia_{l,\pm}^+)|0\rangle, \quad (4a)$$

$$a_{u,\pm}^+|0\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(a_{l,\pm}^+ + ia_{u,\pm}^+)|0\rangle, \quad (4b)$$

where l and u denote the lower optical path and the upper optical path, respectively. $a_{l,\pm}^+|0\rangle$ and $a_{u,\pm}^+|0\rangle$ denote two photons, and \pm denotes the direction of polarization.

From Eq.(4) we found that there will be a $\frac{\pi}{2}$ phase shift between the input photon and the reflected wave function, and the transparent part is synchronized with the input photon. The BS takes no effect on the polarization of the input photon. Because the ions 1, 2 have been placed on the two optical pathes (u and l , respectively), the ions will interact with the photon. The interaction will generate a shift of interference after the BS₂, then Alice will detect the two output ports of the MZI to check whether the teleportation succeeds or not. These are critical to the teleportation and the other quantum information processes to be presented in the current scheme.

Before teleportation the total state of the system is:

$$\begin{aligned} |\Psi\rangle_{total} = & \frac{1}{\sqrt{2}}a_{l,+}^+|0\rangle(\alpha|m_+\rangle_1|m_+\rangle_2|m_+\rangle_3 \\ & + \alpha|m_+\rangle_1|m_-\rangle_2|m_-\rangle_3 \\ & + \beta|m_-\rangle_1|m_+\rangle_2|m_+\rangle_3 \\ & + \beta|m_-\rangle_1|m_-\rangle_2|m_-\rangle_3). \end{aligned} \quad (5)$$

To analyze the evolution of the total system, we will consider the evolution of the following four product states of ions 1, 2:

$$\begin{aligned} a_{l,+}^+|0\rangle|m_+\rangle_1|m_+\rangle_2 \xrightarrow{BS_1, Ions1,2, BS_2} \\ \frac{1}{\sqrt{2}}(|S\rangle_1|g\rangle_1|m_+\rangle_2 + i|m_+\rangle_1|S\rangle_2|g\rangle_2), \end{aligned} \quad (6a)$$

$$\begin{aligned} a_{l,+}^+|0\rangle|m_+\rangle_1|m_-\rangle_2 \xrightarrow{BS_1, Ions1,2, BS_2} \\ \frac{1}{\sqrt{2}}|S\rangle_1|g\rangle_1|m_-\rangle_2 \\ + \frac{i}{2}(a_{u,+}^+ + ia_{l,+}^+)|0\rangle|m_+\rangle_1|m_-\rangle_2 \end{aligned} \quad (6b)$$

$$\begin{aligned} a_{l,+}^+|0\rangle|m_-\rangle_1|m_+\rangle_2 \xrightarrow{BS_1, Ions1,2, BS_2} \\ \frac{i}{\sqrt{2}}|m_-\rangle_1|S\rangle_2|g\rangle_2 \\ + \frac{1}{2}(a_{l,+}^+ + ia_{u,+}^+)|0\rangle|m_-\rangle_1|m_+\rangle_2, \end{aligned} \quad (6c)$$

$$\begin{aligned} a_{l,+}^+|0\rangle|m_-\rangle_1|m_-\rangle_2 \xrightarrow{BS_1, Ions1,2, BS_2} \\ ia_{u,+}^+|0\rangle|m_-\rangle_1|m_-\rangle_2. \end{aligned} \quad (6d)$$

So, after the operation of MZI, the state of the total system will evolve into:

$$\begin{aligned} |\Psi\rangle'_{total} = & \frac{1}{\sqrt{2}}\{\alpha\frac{1}{\sqrt{2}}(|S\rangle_1|g\rangle_1|m_+\rangle_2 \\ & + i|m_+\rangle_1|S\rangle_2|g\rangle_2)|m_+\rangle_3 + \alpha[\frac{1}{\sqrt{2}}|S\rangle_1|g\rangle_1|m_-\rangle_2 \\ & + \frac{i}{2}(a_{u,+}^+ + ia_{l,+}^+)|0\rangle|m_+\rangle_1|m_-\rangle_2]|m_-\rangle_3 \\ & + \beta[\frac{i}{\sqrt{2}}|m_-\rangle_1|S\rangle_2|g\rangle_2 \\ & + \frac{1}{2}(a_{l,+}^+ + ia_{u,+}^+)|0\rangle|m_-\rangle_1|m_+\rangle_2]|m_+\rangle_3 \\ & + i\beta a_{u,+}^+|0\rangle|m_-\rangle_1|m_-\rangle_2|m_-\rangle_3\}. \end{aligned} \quad (7)$$

If the photon detector at the right lower output port D_l fires, the state of the system collapse into:

$$\begin{aligned} |\Psi\rangle_{123} = & \frac{1}{2\sqrt{2}}(-\alpha|m_+\rangle_1|m_-\rangle_2|m_-\rangle_3 \\ & + \beta|m_-\rangle_1|m_+\rangle_2|m_+\rangle_3). \end{aligned} \quad (8)$$

Then Alice will measure the ions 1, 2 in the basis:

$$|+\rangle = \frac{1}{\sqrt{2}}(|m_+\rangle + |m_-\rangle), \quad (9a)$$

$$|-\rangle = \frac{1}{\sqrt{2}}(|m_+\rangle - |m_-\rangle). \quad (9b)$$

For the results $|+\rangle_1|+\rangle_2$, $|-\rangle_1|-\rangle_2$, the ion 3 will be left in the state $-\alpha|m_-\rangle_3 + \beta|m_+\rangle_3$, so a σ_y operation is needed to transfer the state of ion 3 to the state of ion 1. For the results $|+\rangle_1|-\rangle_2$, $|-\rangle_1|+\rangle_2$, the ion 3 will be left in the state $\alpha|m_-\rangle_3 + \beta|m_+\rangle_3$, so a σ_x operation is needed to transfer the state of ion 3 to the state of ion 1. The total success probability of the teleportation scheme is 1/8. Although the successful probability is smaller than 1.0, the current scheme does not need the joint Bell-state measurement and the complex ion trap techniques, which will simplify the implementation of the scheme. The current scheme for the teleporation of one qubit unknown state can be generalized to the multi-qubit unknown state case in a straight forward way. This will be discussed in more detail in other papers.

III. ENTANGLEMENT CONCENTRATION FOR IONIC ENTANGLED STATE VIA ENTANGLEMENT SWAPPING

Suppose there are three spatially separate users Alice, Bob and Cliff. Alice and Bob all share a pair of nonmaximally entangled ions with Cliff. Alice has access to ion 1, Bob has access to ion 4, and ions 2 and 3 are all at Cliff's location. The entangled state of ions 1, 2 and the entangled state of ions 3, 4 are:

$$|\Phi\rangle_{12} = \alpha|m_+\rangle_1|m_+\rangle_2 + \beta|m_-\rangle_1|m_-\rangle_2, \quad (10a)$$

$$|\Phi\rangle_{34} = a|m_+\rangle_3|m_+\rangle_4 + b|m_-\rangle_3|m_-\rangle_4. \quad (10b)$$

Where α, β, a, b are normalization coefficients, and $|\alpha|^2 + |\beta|^2 = 1$, $|a|^2 + |b|^2 = 1$. Without loss of generality, we can assume that α, β, a, b are all real numbers.

Before swapping, the state of the total system is:

$$\begin{aligned} |\Phi\rangle_{total} = & a_{l,+}^+|0\rangle(\alpha\alpha|m_+\rangle_1|m_+\rangle_2|m_+\rangle_3|m_+\rangle_4 \\ & + \alpha\beta|m_+\rangle_1|m_+\rangle_2|m_-\rangle_3|m_-\rangle_4 \\ & + \beta a|m_-\rangle_1|m_-\rangle_2|m_+\rangle_3|m_+\rangle_4 \\ & + \beta b|m_-\rangle_1|m_-\rangle_2|m_-\rangle_3|m_-\rangle_4). \end{aligned} \quad (11)$$

To construct entanglement between ion 1 and ion 4, Cliff will introduce the MZI as in section II, and put the ions 2, 3 on the upper and lower arm of the MZI, respectively. Then One σ^+ polarized photon will be superimposed on the MZI from the left lower input port. From the evolution in Eq.(6), we can give the evolution caused by the MZI:

$$\begin{aligned}
|\Psi\rangle'_{total} = & \alpha a \frac{1}{\sqrt{2}} (|S\rangle_2 |g\rangle_2 |m_+\rangle_3 \\
& + i|m_+\rangle_2 |S\rangle_3 |g\rangle_3) |m_+\rangle_1 |m_+\rangle_4 + \alpha b \left[\frac{1}{\sqrt{2}} |S\rangle_2 |g\rangle_2 |m_-\rangle_3 \right. \\
& + \frac{i}{2} (a_{u,+}^+ + i a_{l,+}^+) |0\rangle |m_+\rangle_2 |m_-\rangle_3] |m_+\rangle_1 |m_-\rangle_4 \\
& + \beta a \left[\frac{i}{\sqrt{2}} |m_-\rangle_2 |S\rangle_3 |g\rangle_3 \right. \\
& + \frac{1}{2} (a_{l,+}^+ + i a_{u,+}^+) |0\rangle |m_-\rangle_2 |m_+\rangle_3] |m_-\rangle_1 |m_+\rangle_4 \\
& \left. + i \beta b a_{u,+}^+ |0\rangle |m_-\rangle_2 |m_-\rangle_3 |m_-\rangle_1 |m_-\rangle_4 \right]. \quad (12)
\end{aligned}$$

Then Cliff will detect the two output ports of the MZI. If the photon detector at the right lower output port D_l fires, the state of the total system will collapse into:

$$\begin{aligned}
|\Phi\rangle_{1234} = & \frac{1}{2} (-\alpha b |m_+\rangle_1 |m_+\rangle_2 |m_-\rangle_3 |m_-\rangle_4 \\
& + \beta a |m_-\rangle_1 |m_-\rangle_2 |m_+\rangle_3 |m_+\rangle_4). \quad (13)
\end{aligned}$$

If we let $\alpha = a$ $\beta = b$, the state in Eq.(13) becomes:

$$\begin{aligned}
|\Phi\rangle_{1234} = & \frac{1}{2} ab (|m_-\rangle_1 |m_-\rangle_2 |m_+\rangle_3 |m_+\rangle_4 \\
& - |m_+\rangle_1 |m_+\rangle_2 |m_-\rangle_3 |m_-\rangle_4), \quad (14)
\end{aligned}$$

which is a four-ion maximally entangled states. If Cliff measures the ions 2, 3 in the $|\pm\rangle$ basis as in section II, the ions 1, 4 will be left in maximally entangled state $\frac{1}{\sqrt{2}}(|m_-\rangle_1 |m_+\rangle_4 - |m_+\rangle_1 |m_-\rangle_4)$ for the results $|+\rangle_2 |+\rangle_3$, $|-\rangle_2 |-\rangle_3$. If the results are $|+\rangle_2 |-\rangle_3$, $|-\rangle_2 |+\rangle_3$, the ions 1, 4 will be left in the maximally entangled state $\frac{1}{\sqrt{2}}(|m_-\rangle_1 |m_+\rangle_4 + |m_+\rangle_1 |m_-\rangle_4)$. The total success probability is $\frac{1}{2}|a|^2(1 - |a|^2)$.

So the two ions 1, 4, which have never interacted before, are left in an entangled state via entanglement swapping. Furthermore, the initial nonmaximally entangled states in Eq.(10) have been concentrated into a maximally entangled state via entanglement swapping [18]. In our previous contributions [13, 14], we also realized the entanglement concentration via entanglement swapping in cavity QED, and the success probabilities of them are bigger than that of the current scheme. But, the current scheme uses the linear optical elements, which can be realized within the current technology easily. In addition, four-ion maximally entangled states can be generated in the current scheme. In one of our previous contributions, we used the similar setup, and realized the purification and concentration of nonmaximally entangled ionic states [15]. But two MZIs have been used there. In

our current scheme, the concentration can be realized by using the MZI only once. So the current scheme is more efficient than the previous one.

IV. REMOTE PREPARATION OF ENTANGLED STATES

In section III, if Cliff want to prepare an entangled state:

$$|\Phi\rangle_{14} = m|m_+\rangle_1 |m_-\rangle_4 + n|m_-\rangle_1 |m_+\rangle_4, \quad (15)$$

on ions 1, 4 remotely, Cliff will measure the state of ions 2, 3 in Eq. (14) in the basis $|\pm'\rangle$:

$$|+\rangle' = \nu|m_+\rangle + \mu|m_-\rangle, \quad (16a)$$

$$|-\rangle' = -\mu|m_+\rangle + \nu|m_-\rangle. \quad (16b)$$

Where $m = \frac{\nu^2}{\sqrt{\nu^4 + \mu^4}}$, $n = \frac{\mu^2}{\sqrt{\nu^4 + \mu^4}}$. After the measurement, the ions 1, 4 will be left in different states corresponding to different measurement results. For results $|+\rangle'_2 |+\rangle'_3$, $|-\rangle'_2 |-\rangle'_3$, ions 1, 4 will be left in maximally entangled state $\frac{1}{\sqrt{2}}(|m_-\rangle_1 |m_+\rangle_4 - |m_+\rangle_1 |m_-\rangle_4)$ with probability $\frac{1}{2}|\mu|^2|\nu|^2|a|^2|b|^2$. For the result $|+\rangle'_2 |-\rangle'_3$, ions 1, 4 will be left in the state in Eq.(15), i.e. the state Cliff want to prepare remotely. The probability is $\frac{1}{4}|a|^2|b|^2(|\mu|^4 + |\nu|^4)$. For the results $|-\rangle'_2 |+\rangle'_3$, ions 1, 4 will be left in the state $\frac{1}{2}ab(\mu^2|m_+\rangle_1 |m_-\rangle_4 + \nu^2|m_-\rangle_1 |m_+\rangle_4)$ with probability $\frac{1}{4}|a|^2|b|^2(|\mu|^4 + |\nu|^4)$. This state can be converted into the state in Eq.(15) by operating σ_x operations on the two ions. So if ions 2, 3 are measured in states $|+\rangle'_2 |-\rangle'_3$, $|-\rangle'_2 |+\rangle'_3$, the ions 1, 4 will be prepared in the state in Eq.(15) successfully with probability $\frac{1}{2}|a|^2|b|^2(|\mu|^4 + |\nu|^4)$.

Compared with the previous scheme for the remote preparation of entangled states, the current scheme embeds the following advantages: (1) it does not need the joint measurement; (2) it can realize the remote preparation of entangled state for ions by using linear optical elements. So it is simpler than the cavity QED or ion-trap schemes.

By far we have only discussed the idea case where we suppose that a photon impinging on an atom always leads to the process described in Eq.(1). But in most case the photon will not be scattered by the ions, if the ions are placed inside the MZI. That would mean that detector at the right upper output port D_u will most likely fire. To enhance the scattering rate, an optical cavity will be added to enclose the MZI. The detailed description has been discussed in Ref. [19], which indicates that this cavity will increase the success probability.

Then we will consider the feasibility of the current scheme. As discussed in Refs. [15, 20], we can use $^{40}\text{Ca}^+$ as the candidate ion for a possible implementation of the

current scheme. $D_{5/2}$ and $D_{3/2}$ are two metastable levels of $^{40}\text{Ca}^+$ with lifetimes of the order of $1s$. s_1 and s_2 are two sublevels of $D_{5/2}$ with $m = -5/2$ and $m = -1/2$, and this two sublevels are coupled to $|e\rangle$ by σ_- and σ_+ light at $854nm$. Here $e, S_1, S_2, S_{1/2}$ correspond to e, m_-, m_+, g in Fig.1 respectively. That is to say, we use the $S_{1/2}$ as stable ground state, S_1, S_2 as two degenerate metastable state and $P_{3/2}$ as excited state. Arbitrary superposition state of this two degenerate metastable states can be realized by applying a laser pulse of appropriate length, which can be realized in a few microsecond [21]. The $^{40}\text{Ca}^+$ in state S_1 or S_2 can be excited into the excited state $P_{3/2}$ by applying one σ_- or σ_+ light at $854nm$. Then decay from $|e\rangle$ to S_1, S_2 , to $D_{3/2}$ and to $S_{1/2}$ are all possible. But Refs. [20, 22] give the transition probability for $P_{1/2} \rightarrow S_{1/2}(397nm)$ as $1.3 \times 10^8/s$ and the branching ratio of $P_{1/2} \rightarrow D_{3/2}(866nm)$ versus $P_{1/2} \rightarrow S_{1/2}(397nm)$ as 1:15, while the branching ratio for $P_{3/2} \rightarrow D_{5/2}(854nm)$ versus $P_{3/2} \rightarrow S_{1/2}(393nm)$ can be estimated as 1:30, giving $0.5 \times 10^7/s$ for the transition probability. So in most case, the $^{40}\text{Ca}^+$ in the excited state will decay into the stable ground state $S_{1/2}$. The detection of the internal states of $^{40}\text{Ca}^+$ can be realized by using a cycling transition between $S_{1/2}$ and $P_{1/2}(397nm)$ [22, 23].

To enhance the emission efficiency of the photons from the ions, we can introduce cavities. Then the following three items will affect the emission efficiency of the photon from the ions:

- The coupling between cavity mode and the $P_{3/2} \rightarrow S_{1/2}(393nm)$ transition;
- Decay from $P_{3/2}$ to $D_{5/2}$;
- Cavity decay.

From reference [24], the probability p_{cav} for a photon to be emitted into the cavity mode after excitation to e can be expressed as $p_{cav} = \frac{4\gamma\Omega^2}{(\gamma+\Gamma)(\gamma\Gamma+4\Omega^2)}$. where $\gamma = 4\pi c/F_{cav}L$ is the decay rate of the cavity, F_{cav} its finesse, L its length, $\Omega = \frac{D}{\hbar} \sqrt{\frac{\hbar c}{2\epsilon_0 \lambda V}}$ is the coupling constant between the transition and the cavity mode, D the dipole element, λ the wavelength of the transition, V the mode volume (which can be made as small as $L^2\lambda/4$ for a confocal cavity with waist $\sqrt{L\lambda/\pi}$), and Γ is the non-cavity related loss rate [20]. From the discussion of Ref. [20], the photon package is about $100ns$, which is a relative long time for the current scheme to be completed.

When calculating the total efficiency of the current scheme (we discuss the entanglement concentration via

swapping as example), we must consider the following items:

- The emission efficiency of photon: p_{cav} , which has included the cavity decay; To maximize the p_{cav} , we have chosen $F_{cav} = 19000$, $L = 3mm$. Then $\gamma = 9.9 \times 10^6/s$, $p_{cav} = 0.01$ [20];
- The effect of the photon detectors is expressed as η . Here we let a detection efficiency $\eta = 0.7$, which is a level that can be reached within the current technology.
- Coupling the photon out of the cavity will introduce another error expressed as ξ , which can be modulated to be close to unit.

In addition, we suppose Alice and Bob all have shared an ensemble of nonmaximally entangled pairs of ions with Cliff. After considering the above factors, the total success probability can be expressed as follow: $P = \frac{a^2(1-a^2)}{2} \times p_{cav}^2 \times \eta \times \xi$ for the entanglement concentration via swapping, that is to say, if we input photon with the rate of $1000000/s$, we can get eight pairs of concentrated entangled $^{40}\text{Ca}^+$ ions per second for $a^2 = 0.7$.

In conclusion, we proposed a linear optical scheme for the teleportation of unknown ionic states, the entanglement concentration for nonmaximally entangled ionic states via entanglement swapping and the remote preparation of entangled states for ions that have never interacted before. The current scheme does not need the realization of the complex joint measurement, i.e. the joint measurement has been converted into the separate measurements on single photons or ions. Quantum information processing for ions can be realized by using linear optical elements. In addition, the scheme avoids the complexity of the ion-trap schemes. However, the current scheme can not realize the quantum dense coding, because the current scheme can not discriminate the four Bell states conclusively.

Acknowledgments

This work is supported by the Natural Science Foundation of the Education Department of Anhui Province under Grant No: 2004kj005zd and Anhui Provincial Natural Science Foundation under Grant No: 03042401 and the Talent Foundation of Anhui University.

[1] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, & W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).

[2] M. Zukowski, A. Zeilinger, M. A. Horne, and A. K. Ekert, Phys. Rev. Lett. 71, 4287 (1993).

[3] C. H. Bennett, D. P. DiVincenzo, P. W. Shor, J. A.

- Smolin, B. M. Terhal, and W. K. Wootters, Phys. Rev. Lett. 87 077902 (2001).
- [4] J.-M. Liu, Y.-Z. Wang, Phys Lett A 316 159 (2003).
 - [5] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992).
 - [6] S.-B. Zheng, and G.-C. Guo, Phys. Rev. Lett. 85, 2392 (2000).
 - [7] J.-W. Pan, D. Bouwmeester, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. 80, 3891 (1998).
 - [8] Z. Zhao, A.-N. Zhang, Y.-A. Chen, H. Zhang, J.-F. Du, T. Yang, J.-W. Pan, Phys. Rev. Lett. 94, 030501 (2005).
 - [9] L. Ye, and G.-C. Guo, Phys. Rev. A 71, 034304 (2005).
 - [10] D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Nature(London) 390, 575 (1997).
 - [11] M. Riebe, H. Häffner, C. F. Roos, W. Hänsel, J. Benhelm, G. P. T. Lancaster, T. W. Körber, C. Becher, F. Schmidt-Kaler, D. F. V. James & R. Blatt, Nature (London) 429, 734 (2004).
 - [12] M. D. Barrett, J. Chiaverini, T. Schaetz, J. Britton, W. M. Itano, J. D. Jost, E. Knill, C. Langer, D. Leibfried, R. Ozeri & D. J. Wineland, Nature (London) 429, 737 (2004).
 - [13] M. Yang, Y. Zhao, W. Song, and Z.-L. Cao, Phys. Rev. A 71, 044302 (2005).
 - [14] M. Yang, W. Song, and Z.-L. Cao, Phys. Rev. A 71, 034312 (2005).
 - [15] M. Yang, W. Song, and Z.-L. Cao, Phys. Rev. A 71, 012308 (2005).
 - [16] X. Zhou, Z.-W. Zhou, M. J. Feldman, and G.-C. Guo, Phys. Rev. A 64, 044103 (2001). X. Zhou, Z.-W. Zhou, G.-C. Guo, and M. J. Feldman, Phys. Rev. A, 64, 020101(R) (2001).
 - [17] P. W. H. Pinkse, T. Fischer, T. P. Maunz, & G. Rempe, Nature(London) 404, 365 (2000). J. Ye, D. W. Vernooy, & H. J. Kimble, Phys. Rev. Lett. 83, 4987 (1999).
 - [18] S. Bose, V. Vedral, and P. L. Knight, Phys. Rev. A 60, 194 (1999).
 - [19] M. Yang, Z.-L. Cao, quant-ph/0312215.
 - [20] C. Simon, and W. T. M. Irvine, Phys. Rev. Lett. 91, 110405 (2003).
 - [21] F. Schmidt-Kaler, S. Gulde, M. Riebe, T. Deuschle, A. Kreuter, G. Lancaster, C. Becher, J. Eschner, H. Häffner, and R. Blatt, J. Phys. B 36, 623 (2003).
 - [22] C. Roos, T. Zeiger, H. Rohde, H. C. Nägerl, J. Eschner, D. Leibfried, F. Schmidt-Kaler, and R. Blatt, Phys. Rev. Lett. 83, 4713 (1999).
 - [23] F. Schmidt-Kaler, H. Häffner, M. Riebe, S. Gulde, GPT Lancaster, T. Deuschle, C. Becher, CF Roos, J. Eschner, and R. Blatt, Nature (London) 422, 408 (2003).
 - [24] S. Haroche and J.-M. Raimond, Adv. At. Mol. Phys. 20, 350 (1985).